Squeezed condensate of gluons and $\eta - \eta'$ mass difference

D. Blaschke [a] H.-P. Pavel [b,1] V.N. Pervushin [c,2] G. Röpke [a,c] and M.K. Volkov [c,2]

[a] *MPG Arbeitsgruppe "Theoretische Vielteilchenphysik" Universität Rostock, D-18051 Rostock, Germany*

[b] *Fachbereich Physik, Universität Rostock, D-18051 Rostock, Germany*

[c] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, 141980, Dubna, Russia*

MPG–VT–UR 89/96
November 1996

Abstract

We consider a mechanism to create the $\eta - \eta'$ mass difference by the gluon anomaly in a squeezed vacuum. We find that the mass shift of the η_0 governing this mass difference is determined by the magnetic part of the gluon condensate. For the squeezed vacuum this magnetic part coincides with the total gluon condensate, so that we get a relation between the gluon condensate and the mass shift of the η_0 as a function of the strong coupling constant α_s. The values of the gluon condensate obtained through this relation are compared with the value by Shifman, Vainshtein and Zakharov and the recent update values by Narison.

PACS number(s): 12.38.Aw, 12.40.Yx, 14.40.Aq, 14.70.Dj

Key words: Squeezed vacuum, gluon condensate, $U_A(1)$ symmetry breaking

[1] Supported by DFG grant No RO 905/11-1
[2] Supported by RFFI grant No 96-01-01223

1. In this note we point out that under the assumption of a squeezed gluon vacuum a relation between the $\eta - \eta'$ meson mass difference and the gluon condensate can be obtained nonperturbatively. Comparison of the condensate values found through this relation with the value of the gluon condensate by Shifman, Vainshtein and Zakharov (SVZ) [1] and with the update average value by Narison [2] allows us to check the applicability of the model of a squeezed gluon vacuum.

Recall that the U(1) problem [3] is the question why the η' mass is much larger than that of the other eight pseudoscalar mesons, especially the η. The $\eta - \eta'$ mass difference $m_{\eta'}^2 - m_\eta^2 = 0.616$ GeV^2 is governed by the mass splitting between the singlet and the octet pseudoscalars η_0 and η_8, which are related to the physical states η, η' via the mixing

$$\begin{aligned} \eta &= \eta_0 \sin\phi - \eta_8 \cos\phi \\ \eta' &= \eta_0 \cos\phi + \eta_8 \sin\phi \; . \end{aligned} \tag{1}$$

A mixing angle $\phi = -(17 \pm 2)^o$ has been obtained in recent analyses of η and η' decays [4,5]. The mass of the η_8 meson as a member of the pseudoscalar flavour octet is well explained by explicit chiral symmetry breaking in accordance with the Goldstone theorem and the Gell-Mann–Oakes–Renner relation. However, explicit chiral symmetry breaking is not sufficient to explain the large mass of η_0 [3].

In the literature [6]-[10] the large mass of η_0 is explained by the gluon anomaly $G^{\mu\nu a}\tilde{G}^a_{\mu\nu} \equiv \partial_\mu K^\mu$. There are several ways to implement this gluon anomaly. In Ref. [6] t'Hooft relates this term to the instanton density in Euclidean space and introduces an effective quark interaction simulating the anomalous term which breaks $U_A(1)$ but conserves the chiral $SU(3)_L \otimes SU(3)_R$ symmetry. This determinant interaction has been widely used within effective quark models such as the NJL model [11,12].

Not using the concept of instantons, other authors [7]-[10] start from an effective hadron Lagrangian which explicitely includes an anomalous meson-gluon interaction term which can be viewed in analogy to the anomalous $\pi^0 \to 2\gamma$ decay

$$\mathcal{L}^{\rm meson}_{\rm singlet} = \frac{1}{2}\partial_\mu\eta_0\partial^\mu\eta_0 - \eta_0\frac{c}{4}G^{\mu\nu a}\tilde{G}^a_{\mu\nu} \; , \tag{2}$$

where

$$c = \sqrt{N_f}\alpha_s/(\pi f_0) \quad , \tag{3}$$

with $f_0/f_\pi \simeq 1$ [4], $f_\pi = 93$ MeV being the pion decay constant and $\alpha_s = g^2/4\pi$ the strong coupling constant. Furthermore, a kinetic term $C(\partial_\mu K^\mu)^2$ is added to (2) and the additional phenomenological constant C is fitted in order to describe the empirical $\eta - \eta'$ mass difference. For a review, see e.g. [13].

2. In difference to [7]-[10] we start from a Lagrangian which includes in addition to (2) the standard gluon kinetic term

$$\mathcal{L}_{\text{singlet}} = \mathcal{L}^{\text{meson}}_{\text{singlet}} - \frac{1}{4} G^{\mu\nu a} G^a_{\mu\nu} \ . \tag{4}$$

From the Lagrangian (4) it should be possible to obtain the mass of the singlet pseudoscalar η_0 as a consequence of the coupling to the gluon field. To calculate the mass of the η_0 let us construct the corresponding Hamiltonian. We have

$$G^{\mu\nu a} G^a_{\mu\nu} = -2(G^a_{0i})^2 + 2(B^a_i)^2 \quad , \quad G^{\mu\nu a} \tilde{G}^a_{\mu\nu} = -4 G^a_{0i} B^a_i \quad , \tag{5}$$

where B^a_i are the components of the magnetic field strength. For the quantization of the physical gluon fields A_0 has to be eliminated, we use here the convention $A_0 = 0$. Introducing the canonical momenta

$$\begin{aligned} \Pi_{\eta_0} &\equiv \frac{\partial \mathcal{L}_{\text{singlet}}}{\partial \dot{\eta}_0} = \dot{\eta}_0 \ , \\ E^a_i &\equiv \frac{\partial \mathcal{L}_{\text{singlet}}}{\partial \dot{A}^a_i} = G^a_{0i} + c B^a_i \eta_0 \ , \end{aligned} \tag{6}$$

the Hamiltonian density reads

$$\begin{aligned} \mathcal{H}_{\text{singlet}} &= \dot{\eta}_0 \Pi_{\eta_0} + \dot{A}^a_i E^a_i - \mathcal{L}_{\text{singlet}} \\ &= \frac{1}{2}\Pi^2_{\eta_0} + \frac{1}{2}(\partial_i \eta_0)^2 + \frac{1}{2}(E^a_i - c B^a_i \eta_0)^2 + \frac{1}{2}(B^a_i)^2 \ . \end{aligned} \tag{7}$$

We would like to eliminate the gluon degrees of freedom by averaging the corresponding Hamiltonian $H = \int d^3x \mathcal{H}$ over the gluon vacuum.

3. In order to perform the averaging over the nonperturbative gluon vacuum we shall use the model of the squeezed gluon vacuum. The squeezed condensate of gluons has been investigated recently [14]-[18] in order to construct a Lorentz and gauge invariant stable QCD vacuum in Minkowski space. Different alternative approaches have not solved this problem. For instance the simple perturbative vacuum is unstable [19], and there is no stable (gauge invariant) coherent vacuum in Minkowski space [20].

From the physical point of view, the squeezed state differs from the coherent one by the condensation of colour singlet gluon pairs rather than of single gluons. In analogy to the Bogoliubov model [21] we consider the case of a homogeneous condensate, but in a squeezed instead of a coherent state. The squeezed vacuum $|0_{\rm sq}[f_0]>$ as a candidate for a homogeneous colourless gluon vacuum is constructed from a nonperturbative reference vacuum $|0> \;\equiv\; |0_{\rm sq}[f_0=0]>$, further specified below, according to

$$|0_{\rm sq}[f_0]> = \; U_{\rm sq}^{-1}[f_0]|0> \;\; . \tag{8}$$

The squeezing operator

$$U_{\rm sq}[f_0] = \exp\left[i\frac{f_0}{2}\sum_{a,i}(A_i^a(0)E_i^a(0)+E_i^a(0)A_i^a(0))\right] \tag{9}$$

with the zero momentum components $A_i^a(0)$ and $E_i^a(0)$ of the fields and their canonical momenta contains the parameter f_0 given below. This special transformation for the homogeneous condensate does not violate Lorentz invariance, since the gauge fields are massless [22]. The question of gauge invariance is very difficult but as in Ref. [23] we suppose the gauge invariance of all the spatial zero momentum components of the gauge fields. The multiplicative transformations of fields corresponding to (8) and (9) are

$$\begin{aligned} U_{\rm sq}[f_0]\; A_i^a(0)\; U_{\rm sq}^{-1}[f_0] &= {\rm e}^{f_0} A_i^a(0)\;, \\ U_{\rm sq}[f_0]\; E_i^a(0)\; U_{\rm sq}^{-1}[f_0] &= {\rm e}^{-f_0} E_i^a(0)\;. \end{aligned} \tag{10}$$

After this canonical transformation the squeezed expectation values as functions of the squeezing parameter f_0 behave like

$$< 0_{\rm sq}[f_0]|\left(B_i^a(0)\right)^2|0_{\rm sq}[f_0]> = e^{4f_0} < 0|\left(B_i^a(0)\right)^2|0> \;\; , \tag{11}$$

$$< 0_{\rm sq}[f_0]|\left(E_i^a(0)\right)^2|0_{\rm sq}[f_0]> = e^{-2f_0} < 0|\left(E_i^a(0)\right)^2|0> \;\; , \tag{12}$$

$$< 0_{\rm sq}[f_0]|E_i^a(0)B_i^a(0)|0_{\rm sq}[f_0]> = e^{f_0} < 0|E_i^a(0)B_i^a(0)|0> \;\; , \tag{13}$$

which follows from (10), noting that $B_i^a(0) = f^{abc}\epsilon_{ijk}A_j^b(0)A_k^c(0)$. Let the reference vacuum $|0>$ be such that the expectation values $< 0|\left(B_i^a(0)\right)^2|0>$, $< 0|\left(E_i^a(0)\right)^2|0>$ and $< 0|E_i^a(0)B_i^a(0)|0>$ behave in the large volume limit $(V\to\infty)$ like $V^{-4/3}$ in accordance with dimensional analysis. The parameter of the squeezing transformation f_0 can be chosen so that the magnetic condensate density (11) remains finite in the large volume limit ($e^{4f_0}\sim V^{4/3}$).

This entails that the electric condensate (12) and the mixed condensate (13) vanish in this limit

$$\lim_{V\to\infty} < 0_{\rm sq}[f_0]|\left(E_i^a(0)\right)^2|0_{\rm sq}[f_0]> \; = \mathcal{O}[1/V^2] \;, \tag{14}$$

$$\lim_{V\to\infty} < 0_{\rm sq}[f_0]|E_i^a(0)B_i^a(0)|0_{\rm sq}[f_0]> \; = \mathcal{O}[1/V] \;. \tag{15}$$

Hence we conclude that in the squeezed vacuum the gluon condensate is equal to its magnetic part,

$$\begin{aligned} < \alpha_s G^2 > \equiv & < 0_{\rm sq}[f_0]|\alpha_s G^{\mu\nu a}G^a_{\mu\nu}|0_{\rm sq}[f_0]> \\ = & \, 2 \; < 0_{\rm sq}[f_0]|\alpha_s (B_i^a)^2|0_{\rm sq}[f_0]> \;. \end{aligned} \tag{16}$$

With these expressions for the averages of the relavant gluon field operators in hand we shall go on in the next paragraph to derive an effective Hamiltonian for the η_0 in the gluon vacuum.

4. Taking the expectation value of the Hamiltonian corresponding to (7) with respect to the squeezed gluon vacuum $|0_{\rm sq}[f_0]>$ we obtain the effective Hamiltonian

$$H_{\rm eff} = \int d^3x \left[\frac{1}{2}\Pi^2_{\eta_0} + \frac{1}{2}(\partial_i\eta_0)^2 + \frac{1}{2}\Delta m_0^2\eta_0^2\right] + {\rm const} \;. \tag{17}$$

with the η_0 mass

$$\Delta m_0^2 = \frac{3\alpha_s}{\pi^2 f_\pi^2} < 0_{\rm sq}[f_0]|\alpha_s(B_i^a)^2|0_{\rm sq}[f_0]> \;. \tag{18}$$

Note that in the effective η_0 Hamiltonian (17) a term linear in η_0 does not appear, since it is proportional to $< 0_{\rm sq}[f_0]|E_i^aB_i^a|0_{\rm sq}[f_0]>$ which according to (15) vanishes in the squeezed vacuum. Because of (16) we can rewrite the mass formula (18) in the form

$$< \alpha_s G^2 > = \frac{2\pi^2 f_\pi^2}{3\alpha_s}\Delta m_0^2 \;. \tag{19}$$

This formula is the main result of our investigation. It relates the gluon condensate to the $U_A(1)$ breaking mass shift of the η_0. The value of α_s in the low energy region is not known very well from experiment. The value used by SVZ [1] is $\alpha_s \approx 1$ and that used by Narison [2] in the low energy region is $\alpha_s(1.3\ {\rm GeV}) \simeq 0.64^{+0.36}_{-0.18} \pm 0.02$. In order to check whether relation (19) is in

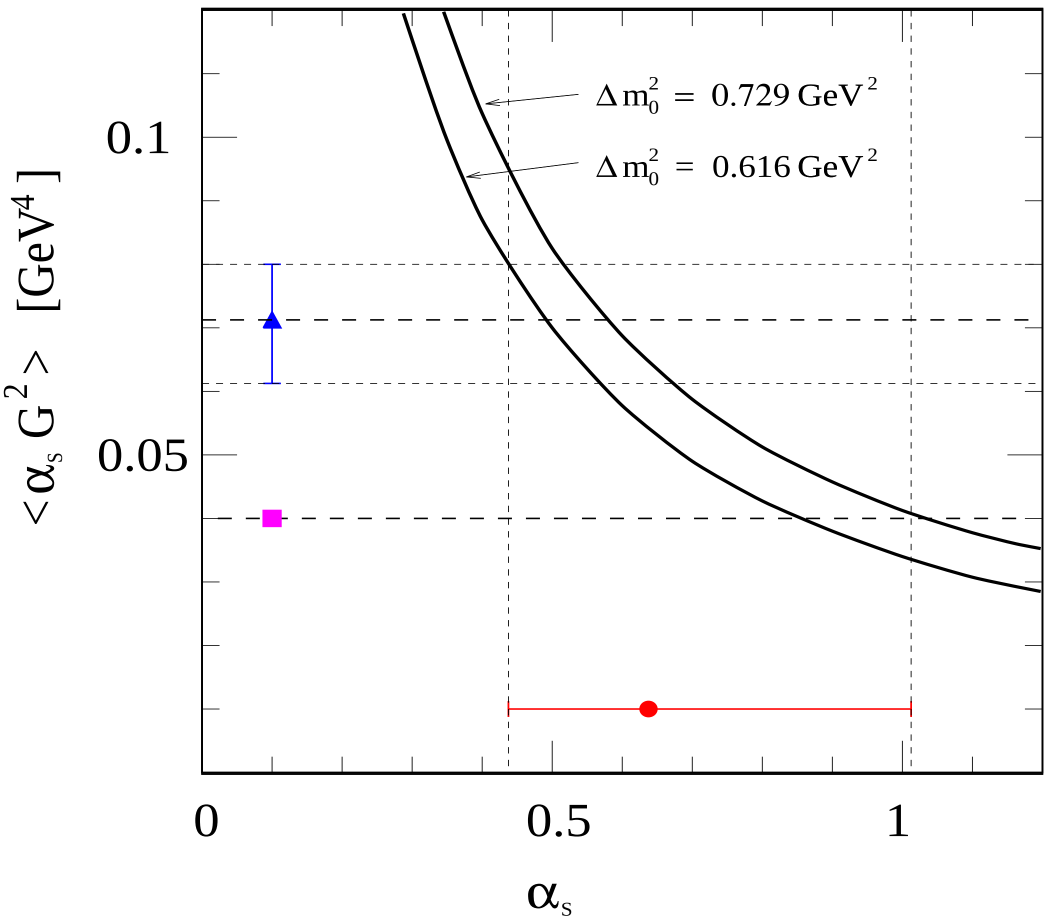

Fig. 1. The gluon condensate $< \alpha_s G^2 >$ vs. QCD coupling α_s for two representative estimates of the η_0 mass shift (solid lines) according to (19). Also shown are the gluon condensate values obtained by Narison [2] (filled triangle) and by Shifman, Vainshtein and Zakharov [1] (filled square) which both are compatible with the $\alpha_s(1.3\text{ GeV})$ value (filled circle) of Ref. [2].

agreement with empirical data we have plotted in Fig. 1 the gluon condensate $< \alpha_s G^2 >$ against α_s for two representative values of Δm_0^2 which are estimates based on the U(3) chiral meson Lagrangian: The first one is obtained for the case of vanishing mixing angle and the chiral limit, where $\Delta m_0^2 = m_{\eta'}^2 - m_\eta^2 = 0.616\text{ GeV}^2$. For the other estimate we use the quadratic Gell-Mann–Okubo mass formula and obtain $\Delta m_0^2 = m_{\eta'}^2 + m_\eta^2 - 2m_K^2 = 0.729\text{ GeV}^2$, see [10]. The solid lines in Fig. 1 give our result for these two typical values of the η_0 mass shift. Also shown are the gluon condensate value $< \alpha_s G^2 > \simeq 0.04\text{ GeV}^4$ by Shifman, Vainshtein and Zakharov [1] (filled square) and the update average value $< \alpha_s G^2 >= (0.071 \pm 0.009)\text{ GeV}^4$ for the gluon condensate obtained by Narison [2] (filled triangle) in a recent analysis of heavy quarkonia mass-splittings in QCD. The gluon condensate values described by our result (19) are in good agreement with both the SVZ and the Narison value for the respective values of α_s in the range of Narison's update average α_s (filled circle). This is an interesting and unexpected result of our investigation.

5. In conclusion we have considered a mechanism to create a large mass for the η_0 due to its anomalous interaction with the gluons of the squeezed vacuum. In the framework of a squeezed vacuum we have obtained a relation between the value of the gluon condensate and the mass shift of the η_0 as a function of the strong coupling constant. The gluon condensate values found in our approach for two estimates of η_0 mass shifts are in quite good agreement with both the "standard" value 0.04 GeV^4 by Shifman, Vainsthein and Zakharov and the update average value 0.071 GeV^4 by Narison for reasonable values of the strong coupling in the low energy region. In the present work we have pointed out a special interesting possibility to resolve the $\mathrm{U}_A(1)$ problem and to obtain estimates for the value of the gluon condensate in the framework of a squeezed gluon vacuum.

Acknowledgement

We are grateful to D. Ebert, A.V. Efremov, E.A. Kuraev and L.N. Lipatov for fruitful discussions. HPP is grateful to the Deutsche Forschungsgemeinschaft for support under contract No. RO 905/11-1. The work of VNP and MKV was supported in part by the RFFI, grant No. 96-01-01223 and the Federal Minister for Research and Technology (BMFT) within Heisenberg-Landau Programme. Two of us (VNP and MKV) acknowledge the financial support provided by the Max-Planck-Gesellschaft and the hospitality of the MPG Arbeitsgruppe "Theoretische Vielteilchenphysik" at the University of Rostock, where part of this work has been done.

Note added: We wish to thank the referee for his comments and for pointing out Ref. [2] to us.

References

[1] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385, 448, 519.

[2] S. Narison, Phys. Lett. B 387 (1996) 162.

[3] S. Weinberg, Phys. Rev. D 11 (1975) 3583.

[4] P. Ball, J.-M. Frère and M. Tytgat, Phys. Lett. B 365 (1996) 367.

[5] T.N. Pham, Phys. Lett. B 246 (1990) 175.

[6] G. 't Hooft, Phys. Rev. D 14 (1976) 3432; 18 (1978) 2199 (E) .

[7] R.J. Crewther, Phys. Lett. 70B (1977) 349.

[8] G. Veneziano, Nucl. Phys. B 159 (1979) 213, P. Di Vecchia and G. Veneziano, Nucl. Phys. B 171 (1980) 253.

[9] C. Rosenzweig, J. Schechter and C.G. Trahern, Phys. Rev. D 21 (1980) 3388.

[10] M.K. Volkov, Sov. J. Part. Nuclei 13 (1982) 446, 17 (1986) 186.

[11] S. Klimt, M. Lutz, U. Vogl and W. Weise, Nucl. Phys. A 516 (1990) 429; S.P. Klevansky, Rev. Mod. Phys. 64 (1992) 649; T. Hatsuda and T. Kunihiro, Phys. Rep. 247 (1994) 221.

[12] V. Dmitrasinović, Phys. Rev. C 53 (1996) 1383 and references therein.

[13] V.A. Miransky, Dynamical Symmetry Breaking in Quantum Field Theories (World Scientific, Singapore, 1993), ch. 12.

[14] L.S. Celenza and C.M. Shakin, Phys. Rev. D 34 (1986) 1591.

[15] I.I. Kogan and A. Kovner, Phys. Rev. D 52 (1995) 3719.

[16] T.S. Biro, Ann. Phys. (NY) 191 (1989) 1; Phys. Lett. B 278 (1992) 15; Int. J. Mod. Phys. 2 (1992) 39.

[17] A. Mishra, H. Mishra, S.P. Misra and S.N. Nayak, Phys. Rev. D 44 (1991) 110; Z. Phys. C 37 (1993) 233; A. Mishra, H. Mishra and S.P. Misra, Z. Phys. C 57 (1993) 241; Z. Phys. C 59 (1993) 159.

[18] V.N. Pervushin, G. Röpke, M.K. Volkov, D. Blaschke, H.-P. Pavel, A. Litvin: Squeezed condensate of Gluons in QCD, Rostock Preprint, MPG-VT-UR 60/96 (1996).

[19] G.K. Savvidy: Phys. Lett. B 71 (1977) 133.

[20] H. Leutwyler: Nucl. Phys. B 179 (1981) 129.

[21] N.N. Bogoliubov, J. Phys. 11 (1947) 23.

[22] A. Linde: Particle Physics and Inflationary Cosmology (Harwood Academic Publishers, Amsterdam, 1990), p. 69.

[23] D. Schütte, Phys. Rev. D 31 (1985) 810.